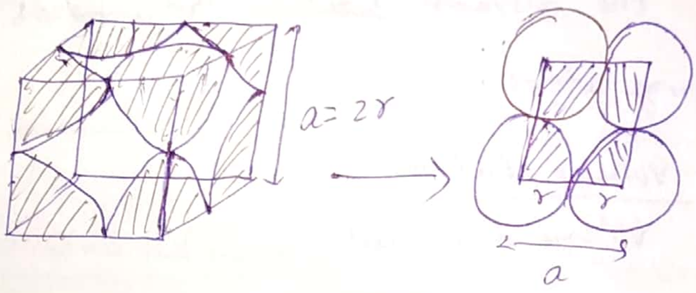
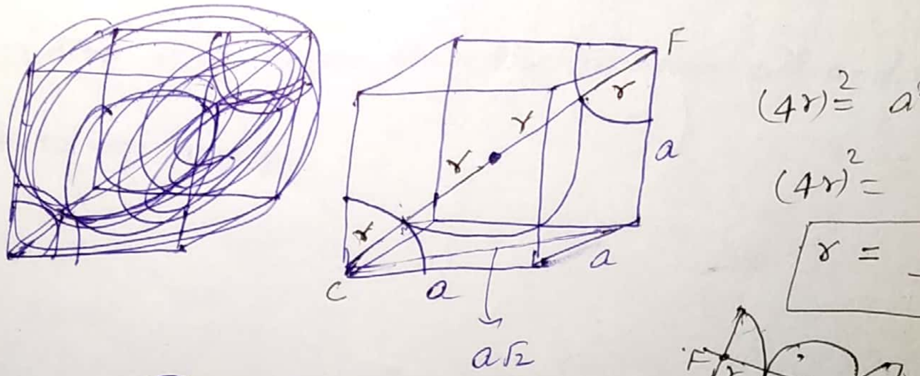
Characteristics of cubic lattice:-

	Simple C	BCC	FCC
Volume, conventional cell	a^3	a^3	a^3
Lattice point per cell	1	2	4
Volume, Primitive cell	a^3	$a^3/2$	$a^3/4$
Lattice point per unit vol.	$1/a^3$	$2/a^3$	$4/a^3$
Number of nearest neighbours	6	8	12
Nearest neighbour distance	a	$\frac{\sqrt{3}a}{2}$	$\frac{a}{\sqrt{2}}$
Number of second neighbours	12	6	6
Second neighbour distance	$\sqrt{2}\,a$	a	a
Packing fraction	$\frac{\pi}{6}=0.524$	$\pi\sqrt{3}/8 = 0.680$	$\pi\sqrt{2}/6 = 0.740$

(a) Simple Cubic



(b) BCC

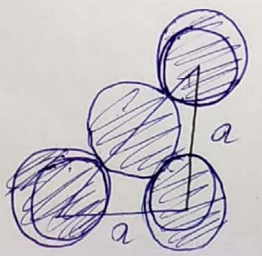


$(4r)^2 = a^2 + (a\sqrt{2})^2$

$(4r)^2 = 3a^2$

$\boxed{r = \frac{a\sqrt{3}}{4}}$

(c) FCC



$\cancel{(a\sqrt{2})^2}\ 2a^2 = (4r)^2 \Rightarrow \boxed{r = \frac{\sqrt{2}a}{4}}$

Hexagonal close Packed (hcp) Structure :-

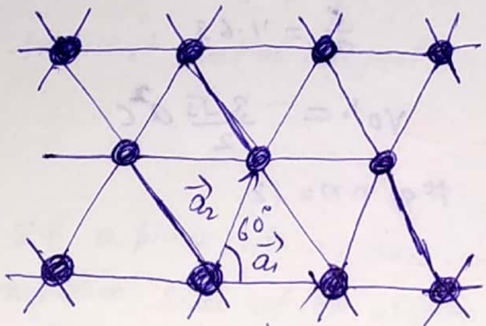
In fcc & hcp packing factor $= 0.74$

ABAB-- Stacking hcp

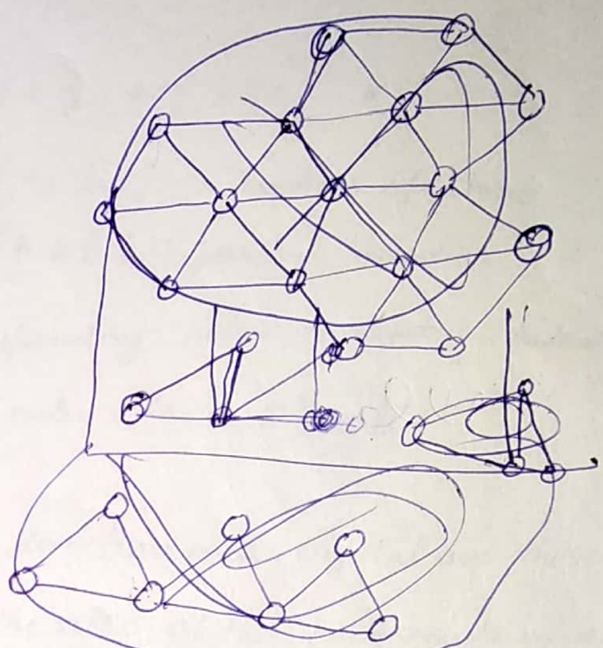
ABCABC-- Stacking fcc

hcp example → Be, cd, O_s ...

First Simple Hexagonal Bravais lattice →



$|\vec{a_1}| = |\vec{a_2}| = a$



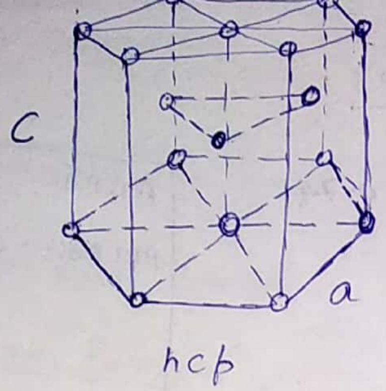
$$\left\{ \begin{array}{l} \vec{a_1} = a\hat{x}, \quad \vec{a_2} = \frac{a}{2}\hat{x} + \frac{\sqrt{3}}{2}a\hat{y} \\ \vec{a_3} = c\hat{z} \end{array} \right.$$

~~defines hex~~

first two primitive vectors generate a triangular lattice in the x-y plane, and the third stacks the planes a distance c above one another.

hcp → two interpenetrating simple hexagonal Bravais lattice displaced from one another by $\frac{\vec{a_1}}{3} + \frac{\vec{a_2}}{3} + \frac{\vec{a_3}}{c}$

Consider stacking cannonballs, starting with a close-packed triangular lattice as the first layer. The next layer is formed by placing a ball in the depression left in the centre of every other triangle in the first layer, forming second triangular layer shifted w.r.t. first. Third layer formed by placing balls in alternate depressions in the second layer, so that they lie directly over the balls in the first layer. Forth layer directly on the second layer and so on.



$$n = \frac{3}{2} + \frac{3}{2} + 3 = 6$$

atoms touch each other along the edge of the hexagon

$\Rightarrow \quad a = 2r$

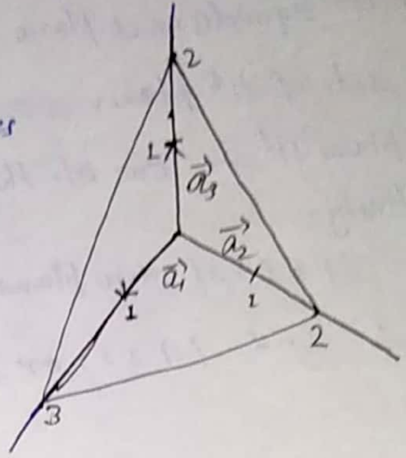
$$\frac{c}{a} = 1.63$$

$$\text{Vol.} = \frac{3\sqrt{3}}{2} a^2 c$$

$\#$ of n.n $= 12$

Miller Indices

(i) Intercepts of Plane on a_1, a_2, a_3 axes are $3a_1, 2a_2, 2a_3$



(ii) $\frac{3a_1}{a_1}, \frac{2a_2}{a_2}, \frac{2a_3}{a_3}$

$\equiv 3, 2, 2$

(iii) Reciprocal $\rightarrow \frac{1}{3}, \frac{1}{2}, \frac{1}{2}$

(iv) smallest set of integral number $6 \times \frac{1}{3}, 6 \times \frac{1}{2}, 6 \times \frac{1}{2}$

$= (2, 3, 3) \rightarrow$ Indices of plane

$(h\,k\,l) \rightarrow$ general notation

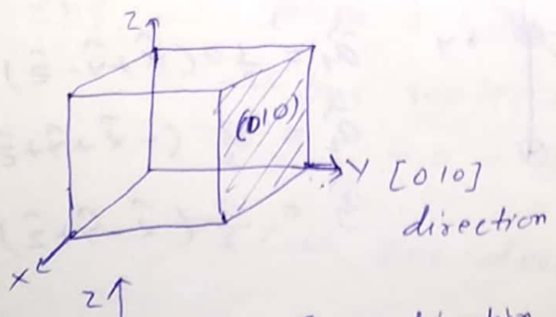
(v) If a plane cuts an axis on the negative side of the origin, the corresponding index is negative, indicated by placing a minus sign above the index: $(h, \bar{k}, l)$ etc.

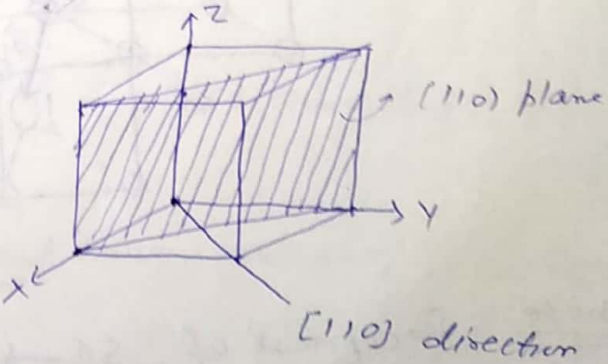
Direction Indices [uvw] of a direction in a crystal are the set of the smallest integers that have the ratio of the components of a vector in the desired direction, referenced to the axes.

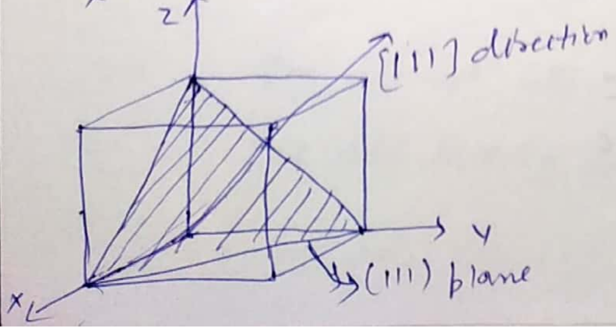
$\vec{a_1}$ axis is [100] direction

$-\vec{a_2}$ axis is $[0\bar{1}0]$ direction.

In Cubic crystal direction [hkl] is $\perp^{ar}$ to a plane (hkl) but this is not generally true in other crystals.







Note (i) All ‖ˡ equidistant plane have same $(h\,k\,l) \Rightarrow (h\,k\,l)$ defines a set of ‖ˡ planes.

(ii) A plane ‖ˡ to one of the coordinate axes has an intercept at infinity.

(iii) If $(h\,k\,l)$ of two planes have the same ratio, i.e, $(8,4\,4)$ and $(4\,2\,2)$ or $(2\,1\,1)$, then planes are ‖ˡ to each other.

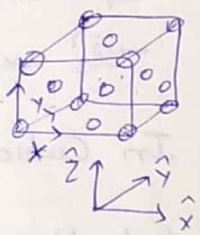
Seperation between lattice planes in Cubic crystal –

$$\boxed{d = \frac{a}{\sqrt{h^2+k^2+l^2}}}$$

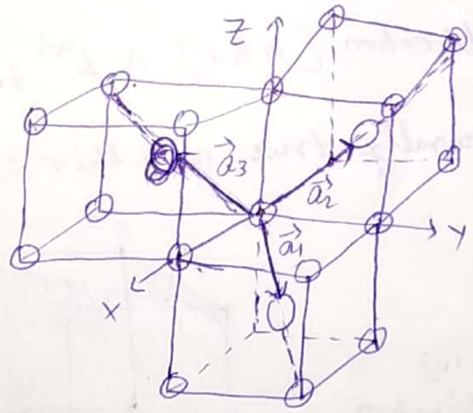
Lattice vectors in Cubic system

SC → $\vec{a}_1 = a\hat{x}$, $\vec{a}_2 = a\hat{y}$, $\vec{a}_3 = a\hat{z}$

fcc → $\vec{a}_1 = \frac{a}{2}(\hat{y}+\hat{z})$, $\vec{a}_2 = \frac{a}{2}(\hat{z}+\hat{x})$, $\vec{a}_3 = \frac{a}{2}(\hat{x}+\hat{y})$



bcc →



$\vec{a}_1 = \frac{1}{2}a(\hat{x}+\hat{y}-\hat{z})$

$\vec{a}_2 = \frac{a}{2}(-\hat{x}+\hat{y}+\hat{z})$

$\vec{a}_3 = \frac{a}{2}(\hat{x}-\hat{y}+\hat{z})$

note Reciprocal of SC → SC
bcc → fcc
fcc → bcc